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ANALYTIC STUDY OF GRID STAR AND REFERENCE STAR SELECTION FOR THE *SPACE INTERFEROMETRY MISSION*

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ABSTRACT

Grid stars and reference stars provide the fundamental global and local astrometric reference frames for observations by the *Space Interferometry Mission*. They must therefore be astrometrically stable at the ~ 1 μas level. I present simple formulae in closed form to estimate the contamination of these frames by motions due to stellar companions that go undetected in a radial-velocity (RV) survey of specified precision. The contamination rate depends almost entirely on the binary-period distribution function and not on the details of the mass or eccentricity distributions. Screening by a modest RV survey ($\sigma_{\rm RV} = 60$ m s^{-1}) can reduce the fraction of grid stars with detectable unmodeled accelerations to $\ll 1\%$. Reference-star selection promises to be much more challenging, partly because the requirements of astrometric stability are more severe and partly because the required density of sources is ~ 100 times higher, the latter of which implies that less satisfactory candidates will have to be accepted. The tools presented here can help design a reference-star selection strategy, but a full treatment of the problem will require better knowledge of their companions in the planetary mass range.

Subject headings: astrometry — methods: statistical — planetary systems

1. INTRODUCTION

By their nature, astrometric observations measure the angular separation of one star relative to another, either absolutely or projected on some axis. Hence, if attention is restricted only to these two stars, one cannot determine which, if either, is stationary and which is moving.

There are basically three approaches to resolving this ambiguity. The traditional method is to find reference stars that, based on their photometric properties, are believed to be so far away that their astrometric motions are small and can be estimated analytically, at least statistically. The *Hipparcos* mission pioneered a second, radically different approach. *Hipparcos* measured the angles between all pairs of stars separated by about a radian. Because each star's position was measured against many others (which lay in sections of the sky where the parallactic motion was substantially different), it was then possible to determine each star's motion individually (up to a global rotation) from a global solution of these measurements. The global rotation was then measured relative to the inertial quasar frame by making use of radio stars whose positions were known in both systems. The *Full-Sky Astrometric Mapping Explorer* (*FAME*) and *Global Astrometric Interferometer for Astrophysics* (*GAIA*) missions also plan to employ this method, except that the quasar tie-in will be done directly in the optical. The *Space Interferometry Mission* (*SIM*) will use a third approach, which is a hybrid of the previous two. All *SIM* astrometry will be done relative to two special classes of stars, "grid stars" and "reference stars."

The grid will be composed of 1000–3000 astrometrically stable stars spread over the whole sky. By repeatedly measuring the relative separations of these stars over the $T_m = 5$ yr span of the mission to ~ 10 μas precision, their absolute parallaxes, relative positions, and proper motions can be determined to $\lesssim 3$ μas, $\lesssim 2$ μas, and $\lesssim 2$ μas yr^{-1}, respectively. The resulting *SIM* frame can be pinned to the extragalactic inertial frame by observing a few quasars (Danner, Unwin, & Allen 1999).[1] Absolute parallaxes, proper motions, and positions of all other objects can then be determined by measuring their positions relative to the grid. For this to work, grid stars must be astrometrically stable on the scale of a few μas: if too many of them prove unstable and "drop out" of the grid, the global grid solution could be undermined. D. Fischer (1998, private communication) originally argued that metal-poor K giants at $D \gtrsim 1$ kpc would make the only truly suitable grid stars because their great distance would reduce astrometric perturbations due to unseen companions. This view has now come to prevail. S. Majewski and collaborators are conducting a photometric search for grid candidates (Patterson et al. 1999). These will ultimately be screened for companions using a radial velocity (RV) survey.

The key problem is to determine in advance what intensity of screening is required to produce a grid that will either have very few drop-outs, or that is structured to be able to "paper over" whatever dropouts it does suffer. This problem has to date been addressed primarily by Monte Carlo simulations (Frink et al. 2001; C. S. Jacobs 2000, private communication; D. Peterson 2001, private communication).

Planet searches with *SIM* will be carried out by measuring target-star positions relative to nearby ($\lesssim 1°$) astrometrically stable (< 1 μas) reference stars. The requirements for reference stars are *qualitatively* similar to those of grid stars, and consequently I will treat the two in parallel. *Quantitatively*, however, treating the reference stars is much more demanding. First, the individual measurement error relative to reference stars must be 1 μas, an order of magnitude more precise than the individual grid measurements. Hence, the stars must correspondingly be more astrometrically stable. Second, reference stars must be brighter to allow for higher precision measurements with a comparable exposure time: grid stars can have $V \sim 12$, but reference

[1] (Danner 1999) is available at http://sim.jpl.nasa.gov/library/book.html.

stars should be 1 or preferably 2 magnitudes brighter. In principle, closer (and hence brighter) stars could be chosen, but, as mentioned above, closer stars are astrometrically less stable. Third, reference stars are required at higher density. I argue in § 5.2 that on the order of eight reference star candidates must be found within ~ 1 deg^2 of the target star, whereas the density of grid stars is $\lesssim 10^{-1}$ deg^{-2}. Thus, the selection of reference stars will likely include objects that are neither metal-poor nor exceptionally luminous, since the density of such rare stars is simply too low. Finally, as I show in § 5.1, grid stars that are accelerating approximately uniformly can perform their grid functions perfectly well. This is not true of reference stars: if their accelerations are at all measurable, then they degrade the sensitivity to outlying planets that are themselves detectable only because of the uniform acceleration that they induce on the targets. Thus, reference-star requirements for astrometric stability are even more demanding compared to grid stars than the relative precisions of measurement would seem to imply. Although the discussion here will be focused on *SIM*, I note that similar considerations apply to the choice of reference stars for planet searches using the Keck Interferometer.

Systematic studies of the reference-star problem are less advanced than those of the grid. G. W. Marcy and M. Shao[2] discuss methods of selecting reference stars and give initial estimates of contamination levels.

Here I present an analytic method for determining the contamination of grid stars and reference stars by undetected stellar companions. This approach is complementary to that of the Monte Carlo studies mentioned above: Monte Carlo methods can model arbitrarily complex distribution functions in arbitrary detail, and so can potentially capture everything that is known about a problem. In the present case, this intrinsic complexity seems formidable, since binary orbits are quite varied and are described by seven parameters. However, the analytic approach has its own advantages. Primarily, it allows one to explicitly see how the results depend on assumptions and on the precision of the measurements. Thus, analytic modeling can be particularly valuable as a guide while the selection criteria and methodology are evolving.

Although the underlying parameter space is quite complicated, I show that the contamination rates can be quickly evaluated using a few simple formulae. In their simplest version, these formulae depend only on the period distribution of the companions and on their mean mass. This version assumes circular orbits, but I also evaluate explicitly the correction due to eccentricities, which is small.

For the most part, only stellar companions are considered in this paper. Brown dwarfs are not a practical concern because of the "brown dwarf desert": the observed lack of brown dwarf companions to G dwarfs (Halbwachs et al. 2000; Marcy & Butler 2000), the progenitors of K giants. Planets are not a major concern for the grid because metal-poor giants are not expected to have many, and because the demand for astrometric stability is not so severe. As I briefly discuss in § 5.2, planets are a significant concern for the reference stars. An analytic treatment of planet contamination is feasible. It would be broadly analogous to the one given here for stellar contamination. However, the data on planetary companions do not currently suffice to use these methods for making reliable predictions of contamination rates. I therefore make reference to such contamination only for purposes of illustration and defer presentation of a detailed treatment.

In § 2, I recapitulate some well-known statistical results that are required for the analysis. In § 3, I analyze the screening of a sample using an RV survey. This analysis applies equally to grid stars and reference stars. In the next two sections on astrometric contamination (§ 4) and implications (§ 5), I treat these two classes in parallel. In § 6, I test the analytic formulae derived here by "predicting" the results of a Monte Carlo simulation by Frink et al. (2001). I find excellent agreement. Finally, in § 7, I summarize the main formulae derived in the paper.

2. STATISTICS PRELUDE

Let $F(t) = \sum_{i=1}^{n} a_i f_i(t)$ be a linear combination of n trial functions $f_i(t)$, each with coefficient a_i. If the parameters a_i are fit to measurements at times t_k, $(k = 1, \ldots, N)$, with errors σ_k, then the covariance matrix $c_{ij} = \mathrm{cov}(a_i, a_j)$ is given by (Press et al. 1992)

$$c = b^{-1}, \quad b_{ij} = \sum_{k=1}^{N} \frac{f_i(t_k) f_j(t_k)}{\sigma_k^2} . \tag{1}$$

For the special case of a two-parameter fit $F(t) = a_1 + a_2 t$, in which the errors are all equal, $\sigma_k = \sigma$, equation (1) implies that the variance of the slope is given by

$$\mathrm{var}(a_2) \equiv c_{22} = \frac{\sigma^2}{N \mathrm{var}(t)} , \tag{2}$$

where $\mathrm{var}(t) \equiv \langle t_k^2 \rangle - \langle t_k \rangle^2$. Finally, if there are a large number of measurements uniformly spaced over an interval $[-T/2, T/2]$, also with equal errors, but allowing for an arbitrary number of parameters, then equation (1) becomes

$$b_{ij} = \frac{N}{\sigma^2 T} \int_{-T/2}^{T/2} dt\, f_i(t) f_j(t) . \tag{3}$$

For a polynomial, $F(t) = \sum_{i=1}^{n} a_i t^{i-1}/(i-1)!$, these components are

$$b_{ij} = \frac{N}{\sigma^2} \frac{(T/2)^{i+j-2}}{(i+j-1)(i-1)!(j-1)!} \quad \text{for } i+j \text{ even} , \tag{4}$$

and $b_{ij} = 0$ otherwise. Hence, even for a cubic ($n = 4$), b decomposes into two (2×2) matrices that are easily inverted by hand.

3. RADIAL VELOCITY SCREENING

I first ask the question: what fraction of an initial sample of candidate grid stars or reference stars will be rejected by an RV survey of N measurements equally spaced over a time T_{RV}, each with a precision σ_{RV}? I assume that the candidates have mass $M = 1\ M_\odot$ and focus initially on stellar companions of mass m, i.e., $0.1 < m < 1\ M_\odot$. Note that for the parameters of interest, $T_{\mathrm{RV}} \sim 5$ yr and $\sigma_{\mathrm{RV}} \sim 60$ m s^{-1}, essentially all companions with periods $P \lesssim 2T_{\mathrm{RV}}$ will easily be detected. For example, the velocity semi-amplitude induced by an $m = 0.1\ M_\odot$ companion in a $P = 2T_{\mathrm{RV}} = 10$ yr orbit is 1.5 km s^{-1}, so that only extremely face-on orbits, $\sin i < 0.08$, would escape detection. Since these constitute about 0.3% of all orientations (and so ~ 0.02% of all candidates), they will be of no interest here.

[2] The *SIM* proposals of G. W. Marcy and M. Shao are available at http://sim.jpl.nasa.gov/ao_support/ao_abstracts.html.

Instead, I will work in the limit of uniform accelerations, the magnitude of whose radial component is given by

$$a_r = \frac{Gm}{r^2}\,|\cos\theta| = \frac{63\ \mathrm{m\ s^{-1}}}{\mathrm{yr}}\left(\frac{m}{0.3\ M_\odot}\right)\left(\frac{r}{30\ \mathrm{AU}}\right)^{-2}|\cos\theta|\,, \tag{5}$$

where r is the instantaneous separation of the companion and θ is the angle between the separation vector and the line of sight.

On the other hand, from equation (2), and using $\sum_{k=1}^{N} k = N(N+1)/2$ and $\sum_{k=1}^{N} k^2 = N(N+1)(2N+1)/6 \Rightarrow \mathrm{var}(k) = (N^2-1)/12$, the RV survey can measure such accelerations with a precision

$$\sigma_a = \sqrt{\frac{12(N-1)}{N(N+1)}}\,\frac{\sigma_{\rm RV}}{T_{\rm RV}} = \frac{16\ \mathrm{m\ s^{-1}}}{\mathrm{yr}}\left(\frac{\sigma_{\rm RV}}{60\ \mathrm{m\ s^{-1}}}\right)\left(\frac{T_{\rm RV}}{5\ \mathrm{yr}}\right)^{-1}\times\left[\frac{N(N+1)/(N-1)}{6.67}\right]^{-1/2}. \tag{6}$$

Here $\sigma_{\rm RV}$ includes contributions from both measurement error and intrinsic velocity variability of the stellar atmosphere, and the expression is normalized to $N=4$ observations. I assume that candidates will be rejected if the observed accelerations exceed a threshold $a_{\rm thr} = 2\sigma_a$: if the threshold were set much lower, too many candidates would be eliminated by statistical noise. For an ensemble of systems, $|\cos\theta|$ is uniformly distributed over [0, 1]. Hence, the fraction f_p that will pass the RV cut is

$$f_p(r, m) = \min\left\{\frac{a_{\rm thr}}{Gm/r^2}\,,\,1\right\} = \min\left\{\frac{2r^2\sigma_a}{Gm}\,,\,1\right\}. \tag{7}$$

An ensemble of companions with semimajor axis A and eccentricity e will be found equally distributed in all phases of their orbits. Hence, the fraction passing the RV screening is

$$f_p(e, A, m) = 1 \quad \text{for}\ \frac{Gm}{[A(1-e)]^2} \le a_{\rm thr}, \tag{8}$$

$$f_p(e, A, m) = \left(1+\frac{3}{2}e^2\right)\frac{A^2 a_{\rm thr}}{Gm} \quad \text{for}\ \frac{Gm}{[A(1+e)]^2} \ge a_{\rm thr}. \tag{9}$$

In the intervening range, $f_p(e, A, m)$ can be written in closed form, but the expression is not illuminating. For the moment, I use the approximation of circular orbits. I address the question of eccentric orbits in § 3.2.

3.1. *Circular Orbits*

The main progenitors of K giants are G dwarfs, whose companion distribution has been studied by Duquennoy & Mayor (1991, hereafter DM91). They find that about 60% of G dwarfs have companions, and that the log periods of these are roughly Gaussian distributed with $\langle \log (P/\mathrm{days})\rangle = 4.85$ and $(\mathrm{var}[\log (P/\mathrm{days})])^{1/2} = 2.3$. The companion mass distribution is roughly flat over the interval $0.1 < m < 1\ M_\odot$. By comparison with this broad distribution over roughly seven decades in semimajor axes, equations (8) and (9) show that f_p rises from nearly zero to 1 in less than a decade. Therefore, it is appropriate to evaluate $f_p(m)$ over all A as being equal to the fraction of candidates with $A > A(m)$, where $f_p(A(m), m) = 0.5$. That is,

$$A(m) = \sqrt{\frac{Gm}{2a_{\rm thr}}} = 30\ \mathrm{AU}\left(\frac{m}{0.3\ M_\odot}\right)^{1/2}\left(\frac{a_{\rm thr}}{32\ \mathrm{m\ s^{-1}\ yr^{-1}}}\right)^{-1/2}. \tag{10}$$

For masses of $m = 0.1$ and $1\ M_\odot$, these values correspond to periods of $P = 10^{4.4}$ and $10^{5.0}$ days, respectively. That is, the entire mass range corresponds to only about half a decade of the DM91 binary distribution function. Hence, to evaluate the rejected fraction $f_{\rm rej,RV}$ over the entire mass range, it is appropriate to simply adopt the period corresponding to the average mass, $\langle m\rangle \sim 0.3\ M_\odot$, which is equivalent to assuming that the DM91 cumulative distribution function is a straight line within a bin. The error so induced is less than 1%, much less than the Poisson noise in the measurement of the distribution function itself. This yields

$$f_{\rm rej,RV} = \int_0^{P_*} dP\,\frac{df_b}{dP}\,,\quad P_*^2 \equiv \frac{4\pi^2[A(\langle m\rangle)]^3}{G(M_\odot + \langle m\rangle)}\,, \tag{11}$$

where df_b/dP is the binary-period distribution function. For the fiducial parameters I have been considering, $P_* = 10^{4.7}$ days. Thus, substituting the period distribution of DM91 into equation (11), and using these fiducial parameters, $f_{\rm rej,RV} = 29\%$ of the candidates would be rejected. In the scenario I have laid out here, another 5% of the remaining 71% would be rejected because statistical fluctuations would cause the stars to appear to accelerate at the 2 σ level, even when there was no real acceleration.

3.2. *Eccentric Orbits*

To include eccentric orbits exactly would be complicated, because the transition from equation (8) to equation (9) is complicated. However, the order of the effect can be assessed by noting that in both these limiting regimes, the change is accounted for by $a_{\rm thr} \to [1 + (3/2)e^2]a_{\rm thr}$, and therefore by making the following substitution in equation (10):

$$a_{\rm thr} \to g_e\, a_{\rm thr}\,,\quad g_e \equiv 1 + \tfrac{3}{2}\langle e^2\rangle\,. \tag{12}$$

Then, following through with the remaining logic in § 3.1, this leads to a change in the estimate of $f_{\rm rej,RV}$,

$$\Delta f_{\rm rej,RV} = -\frac{3}{4}\log g_e \left.\frac{df_b}{d\log P}\right|_{P_*} = -0.02\,\frac{\log g_e}{\log (7/4)}\,\frac{(df_b/d\log P)|_{P_*}}{0.11}\,, \tag{13}$$

where the evaluation has again been made using f_b from DM91. Because this change is so small, I generally assume circular orbits, but include a 2% adjustment for eccentricity.

4. FRACTION OF ASTROMETRIC ACCELERATORS

4.1. *Grid Stars*

Of the candidates that pass the RV screening and go on to become grid stars, what fraction will have accelerations that are detectable astrometrically? I choose a 3 σ criterion to avoid excessive rejection due to noise. The astrometric acceleration α is given by

$$\alpha = \frac{Gm}{Dr^2} \sin\theta$$
$$= \frac{13\ \mu\mathrm{as}}{\mathrm{yr}^2}\left(\frac{m}{0.3\ M_\odot}\right)\left(\frac{r}{30\ \mathrm{AU}}\right)^{-2}\left(\frac{D}{1\ \mathrm{kpc}}\right)^{-1} \sin\theta\ , \quad (14)$$

where D is the distance to the grid star, and all the remaining quantities are the same as in equation (5). Suppose that the astrometric data in one direction are fit to the form $\psi(t) = \psi_0 + \mu_0 t + (1/2)\alpha t^2 + \kappa\Pi \cos[2\pi(t - t_0)/\mathrm{yr}]$, where ψ_0 and μ_0 are the position and proper motion, respectively, at midmission, Π is the parallax, and κ and t_0 give the phase and projection factor of the parallax ellipse. Since the grid will be surveyed many (e.g., $N \sim 23$) times, the covariances of these four parameters can be evaluated using equation (3). First, I note that since the parallax term is cyclic, whereas all the others are secular, $b_{i4}/(b_{ii}b_{44})^{1/2} \ll 1$. Thus, the parallax terms decouple and can be ignored for present purposes. The remaining terms are then given by equation (4), and the resulting covariance matrix is therefore

$$c = \sigma_0^2 \begin{pmatrix} \frac{9}{4} & 0 & -\frac{30}{T_m^2} \\ 0 & \frac{12}{T_m^2} & 0 \\ -\frac{30}{T_m^2} & 0 & \frac{720}{T_m^4} \end{pmatrix}, \quad (15)$$

where T_m is the mission duration and $\sigma_0 = \sigma/N^{1/2}$ is the position error for the case in which the data are fitted to uniform motion. Hence, the acceleration error is

$$\sigma_\alpha = \frac{2\ \mu\mathrm{as}}{\mathrm{yr}^2}\left(\frac{\sigma_0}{2\ \mu\mathrm{as}}\right)\left(\frac{T_m}{5\ \mathrm{yr}}\right)^{-2} . \quad (16)$$

As stated above, the threshold of detection is $\alpha_{\mathrm{thr}} = 3\sigma_\alpha$. We can now define a figure of merit K for the relative sensitivities of the astrometric and RV surveys,

$$K \equiv \frac{a_{\mathrm{thr}}}{D\alpha_{\mathrm{thr}}}$$
$$= 1.13\left(\frac{a_{\mathrm{thr}}}{32\ \mathrm{m\ s^{-1}\ yr^{-1}}}\right)\left(\frac{\alpha_{\mathrm{thr}}}{6\ \mu\mathrm{as\ yr^{-2}}}\right)^{-1}\left(\frac{D}{\mathrm{kpc}}\right)^{-1} . \quad (17)$$

Roughly speaking, this means that the astrometric survey can detect $K = 1.1$ times smaller accelerations than the RV survey, and so can detect companions at $K^{1/2}$ greater separations or $K^{3/4}$ greater periods. Naively, this would appear to mean that the fraction of grid stars with detectable accelerations (relative to the pre-RV sample) is given by

$$f_{\mathrm{acc}} = \frac{3}{4}\frac{df_b}{d\log P}\bigg|_{P_\dagger} G(K)\ , \quad (18)$$

where $P_\dagger \sim P_*$ and

$$G(K) \to G_0(K) = \log K \quad (\mathrm{naive})\ . \quad (19)$$

However, this simple treatment ignores the different θ-dependence in equations (5) and (14): the astrometric survey is sensitive to two components of acceleration, and corresponding to this, the mean value of $\sin^2\theta$ is twice as high as $\cos^2\theta$ over a sphere. In addition, the stars with low $\cos\theta$, which are most likely to evade the RV surveillance, are the most easily detected astrometrically. A more rigorous treatment yields

$$f_{\mathrm{acc}}(m) = \int d\log r\, \frac{df_b}{d\log r} \min\{y, \sqrt{1-(y/K)^2}\},$$
$$\text{where } y(r; m) \equiv \frac{a_{\mathrm{thr}} r^2}{Gm} . \quad (20)$$

Because the second factor differs significantly from zero over only a relatively narrow range, the first factor can be pulled out of the integral, which then yields equation (18), with

$$G(K) = \int d\log y\, \min\{y, \sqrt{1-(y/K)^2}\}$$
$$= \log(K + \sqrt{K^2+1})\ , \quad (21)$$

and with $P_\dagger$ evaluated midway between the astrometric and RV sensitivities,

$$P_\dagger = K^{3/8} P_* = \frac{2\pi K^{3/8}}{\sqrt{G(M_\odot + m)}}\left(\frac{Gm}{2a_{\mathrm{thr}}}\right)^{3/4} . \quad (22)$$

Again, $\log P_\dagger$ varies by only 0.6 over the whole mass range, so it is appropriate to evaluate f_{acc} at $\langle m\rangle = 0.3\ M_\odot$. In this case, the correction for eccentric orbits is completely negligible, because the effect is simply to slightly displace $P_\dagger$. However, for the actual parameters of interest, $P_\dagger$ is near the peak of the DM91 distribution (where $df_b/d\log P = 18/164 \sim 11\%$), so the evaluation of equation (18) does not depend on the exact choice of $P_\dagger$. For the fiducial parameters I have been using, $f_{\mathrm{acc}} = 3.5\%$. Hence, a fraction $f_{\mathrm{acc}}/(1 - f_{\mathrm{rej,RV}}) \sim 5\%$ of the grid stars that survive RV surveillance will have detectable astrometric accelerations. Here I have used $f_{\mathrm{rej,RV}} = 30\%$ to account for both eccentricity and candidates eliminated by statistical fluctuations.

4.2. *Reference Stars*

Much of the foregoing derivation carries through for reference stars. There are three major differences. First, as I discuss in § 5.2, reference stars are likely to be closer: I adopt $D = 600$ pc. Second, the reference stars will be subject to measurements of much higher precision. Typically, target stars are expected to be observed relative to four reference stars at $N \sim 25$ epochs, each time to a total positional precision of $\sigma_{\mathrm{pos}} \sim 1\ \mu$as. Since each reference star is observed only 1/4 of the time, and since their accelerations must be detected against each other, this implies that σ_0 in equations (15) and (16) should be evaluated,

$$\sigma_0 = \frac{\sqrt{8/3}\,\sigma_{\mathrm{pos}}}{\sqrt{N}} = 0.33\ \mu\mathrm{as}\left(\frac{\sigma_{\mathrm{pos}}}{1\ \mu\mathrm{as}}\right)\left(\frac{N}{25}\right)^{-1/2} , \quad (23)$$

which is a factor 6 times smaller than for grid stars. Consequently, K is a factor of 6/0.6 times larger, $K = 11$.

Third, depending on the ultimate design of *SIM*, the reference stars may be measured relative to the target only along one dimension (" ParaSIM ") or in two dimensions

(" shared baseline "). In the latter case, the analysis is identical to that given in § 4.1. Using $K = 11$ and equations (18) and (21), I then find $f_{\rm acc} = 11\%$, implying that a fraction $f_{\rm acc}/(1 - f_{\rm rej,RV}) \sim 16\%$ of reference stars that survive RV surveillance will have measurable accelerations.

For the former case (which at the present time appears less likely), $\sin\theta$ in equation (14) should be replaced by $\sin\theta\,|\cos\phi|$, where ϕ is a random angle on a circle. In principle, this means that equation (20) should be replaced by a more complicated integration. In fact, this is unnecessary for the case $K \gg 1$, which is of relevance here. Since the RV and astrometric measurements are both one-dimensional, the fraction of stars with detectable RV accelerations at radius $r_{\rm RV}$ will be exactly the same as those with detectable astrometric accelerations at $r_{\rm ast} = K^{1/2} r_{\rm RV}$. Hence, if these two detection processes could be regarded as completely independent of one another, equations (18) and (19) would hold. As discussed following equation (19), these detection processes are not generally independent. However, for $K \gg 1$, they are approximately independent because at the radii where the radial acceleration is detectable at all, there is only a very small chance that the astrometric acceleration will be undetectable. Applying equations (18) and (19), and taking $K = 11$, I find $f_{\rm acc} = 9\%$, implying that $f_{\rm acc}/(1 - f_{\rm rej,RV}) \sim 12\%$.

5. IMPLICATIONS

5.1. *Grid Stars*

In § 4.1, I showed that about 5% of *SIM* grid stars that survive RV selection will have measurable astrometric accelerations. If it were necessary to eliminate these stars from the grid well into the mission, then it would be necessary to build redundancy into the grid to " paper over " the resulting holes. In fact, this is not necessary. The fundamental reason is that, as I show below, while some stars surviving RV selection may have detectable accelerations, almost none have detectable jerks. If this is the case, the accelerating stars can be fitted to seven parameters (including two components of acceleration) instead of the usual five. From equation (15), this acceleration measurement decouples completely from the proper-motion measurement, and as discussed directly above equation (15), it decouples from the parallax measurement as well. Thus, the grid parallax and proper motions (the main reasons for having a grid) are not significantly affected by fitting for acceleration. The error in the mean position is increased by a factor of $c_{1,1}^{1/2}/\sigma_0 = 1.5$. However, this error is more than an order of magnitude below any known requirement. For example, the positions are used for the tie-in to the radio reference frame, but the latter is known only to 20 μas, and the individual positions of quasars are much more poorly determined. Moreover, this slight degradation in positional error affects only 5% of the grid stars.

What fraction of grid stars surviving RV selection will have detectable jerks? Noting that the parallax measurement again decouples (see § 4.1), I write the remaining positional dependence in one direction as $\psi(t) = \psi_0 + \mu_0 t + (1/2)\alpha_0 t^2 + (1/6) j t^3$. I then evaluate b_{ij} using equation (3) and invert the resulting (2×2) (proper-motion, jerk) submatrix, thus finding

$$\sigma_j = \sqrt{100{,}800}\,\frac{\sigma_0}{T_m^3} = \frac{5\ \mu\text{as}}{\text{yr}^3}\left(\frac{\sigma_0}{2\ \mu\text{as}}\right)\left(\frac{T_m}{5\ \text{yr}}\right)^{-3} . \quad (24)$$

For circular orbits of period P, the astrometric jerk is given by

$$j = \frac{m}{M_\odot}\,\frac{r}{D}\left(\frac{2\pi}{P}\right)^3 \simeq \frac{8\ \mu\text{as}}{\text{yr}^3}\left(\frac{m}{0.1\ M_\odot}\right)\left(\frac{r}{10\ \text{AU}}\right)^{-7/2}\left(\frac{D}{\text{kpc}}\right)^{-1} , \quad (25)$$

where to be conservative I have assumed that the jerk is in the plane of the sky and have written the formula in a way that is strictly appropriate only for $m \ll M_\odot$. I show below that this is the regime of the greatest concern. Hence, for the fiducial parameters, the jerk is detectable at the 2 σ level, provided that $r < 9\ \text{AU}(m/0.1\ M_\odot)^{2/7}$. In the uniform-acceleration approximation, such accelerations escape RV detection only a fraction $0.14(m/0.1\ M_\odot)^{-3/7}$ of the time (see eqs. [5]–[7]). However, at $r = 9$ AU, the uniform-acceleration approximation is already starting to break down, and as I discussed in § 3, for $r < 4.5$ AU, the RV survey almost never fails to detect stellar companions. I therefore estimate that for 1/2 dex in log P, the RV survey misses $\sim 10\%$ of grid candidates that go on to show marginally detectable jerk, i.e., about 0.4% of grid stars.

To understand the effect of eccentricity, note that the worst case is a companion at periastron r of a highly eccentric orbit. Then the acceleration is the same as for a circular orbit at r, but the jerk is larger by $\sqrt{2}$. Since stars spend little time at periastron, and since the effect itself is small, I ignore it.

Finally, I have ignored the planetary companions of grid candidates. The plan is for the grid to be composed of metal-poor K giants, which are not expected to have planets for several reasons. First, planet frequency among G stars (the progenitors of K giants) in the solar neighborhood is highly correlated with metallicity (Gonzalez 1997; Gonzalez, Wallerstein, & Saar 1999). Second, planets occur in 47 Tucanae at a rate that is much lower than that of solar-metallicity stars, and is consistent with zero (Gilliland et al. 2000). Third, if gas giant planets grow from rock and ice cores, as most current theories suggest, it is difficult to see how they would get started in a metal-poor environment. This optimistic assessment could prove wrong, but if so, it will become evident early in the RV survey of candidates, which would give plenty of time to modify strategy. For now, it is reasonable to suppose that such planets will be rare or nonexistent.

In brief, with the very reasonable fiducial parameters adopted here, I expect that RV screening can identify grid stars whose chance of being corrupted by an undetected companion is $\ll 1\%$.

5.2. *Reference Stars*

As I discussed in the introduction, contamination is a much more severe problem for reference stars than it is for the grid. First, the astrometric measurements need to be substantially more precise, meaning that they are more sensitive to contaminants. Second, the reference stars need to be brighter (to achieve this greater astrometric precision in a short exposure), which generally means that they need to be closer. This in turn increases the astrometric contamination, which scales $\propto D^{-1}$. Third, their surface density on the sky needs to be higher, which means one cannot typically find metal-poor halo stars to serve as reference stars.

Hence, a significant fraction are likely to have planets. Finally, uniform acceleration seriously undermines the function of reference stars, whereas, as we saw in § 5.1, it is perfectly acceptable for the grid. Here I employ the formalism developed above to illuminate these problems.

In order to be sensitive to very low levels of acceleration due to distant planets, the observer must find at least two reference stars[3] that are themselves unaccelerated. This is because if the target star and the reference star are found to be accelerating relative to one another, there is no way to determine which is truly accelerating and which (if either) is in uniform motion. It is only by finding two reference stars that are not accelerating relative to one another that one can have reasonable confidence that it is they, and not the target, that are in uniform motion. If two such stars cannot be found, then one still has sensitivity to planets with periods $P \lesssim T_m$, but not to outlying planets, including outlying companions of planets with short periods (i.e., planetary systems).

The first point then, is that the RV survey must begin with enough candidates to have good prospects (say, a 95% probability) of finding at least two candidates that have no detectable RV acceleration. I showed in § 3 that with a 5 yr, 60 m s^{-1} survey, one could expect 30% rejection because of stellar companions and statistical fluctuations. I have not evaluated planet contamination in this paper, but for illustration, I will assume that an additional 7% of candidates are eliminated by the RV survey because of planets. This means that if one wants to wind up with two reference stars, one should begin with eight candidates.

The second point is that if these are the only reference stars used, then the probability that one of them will prove to be an astrometric accelerator is high. In § 4.2, I showed that 16% would have detectable accelerations because of stellar companions. However, there will be additional losses because of planets. For example, a Jupiter-mass planet at 3 AU would generate a velocity semiamplitude of only 17 sin i m s^{-1}, well below the detection threshold of the RV survey, but would generate an astrometric semiamplitude of 5 μas, which would be quite easily detectable. Hence, an additional 5% loss due to planets is quite plausible. If so, the probability of at least one of the two reference stars being an accelerator would be 38%.

There are basically four alternatives. (1) Accept that for more than a third of the target stars there will be sensitivity to closed planetary orbits, but not to distant planets. (2) Accept fainter reference stars to increase D, and so decrease astrometric contamination, but thereby degrade the astrometric precision and so the sensitivity to low-mass planets. (3) Increase the number of reference stars to increase robustness. (4) Increase surveillance efforts to reduce contaminants.

I have not much to say about the first two options, save that I would be disappointed if they were adopted. The third can be implemented at fairly low cost. If eight candidates are initially surveyed, then there is an 87% chance that at least three will survive the RV survey, and 66% chance that four will survive. With three reference stars instead of two, the probability that at least two will survive rises from 62% to 89%. Of course, there is then also a high additional probability (39%) that one of these three will fail, in which case the errors for detecting distant planets (but not for close ones) would increase by $(3/2)^{1/2}$. This is still a lot better than a total loss. The situation would be even more favorable with four reference stars.

Finally, a more intensive RV study would be expensive in terms of big-telescope time, but would be effective against both stellar and planetary companions. For example, reducing $\sigma_{\rm RV}$ from 60 to 20 m s^{-1} would decrease the number of stellar contaminants by $\Delta \ln G(K) \sim 36\%$. It would not be possible to go below 20 m s^{-1}, since this is the typical scale of photospheric fluctuations of K giants (Frink et al. 2001). However, if need be, one could achieve the same effect with multiple measurements (assuming, as is almost certainly the case, that these fluctuations overwhelmingly have power on short timescales, so that they do not couple to the acceleration measurements).

6. COMPARISON WITH MONTE CARLO

Comparison with the Monte Carlo simulation of Frink et al. (2001) permits a direct check of the foregoing calculation, but first it is necessary to translate their parameterization into the one used here. They consider an RV survey with $N = 2$ epochs separated by $T_{\rm RV} = 5$ yr, each with $\sigma_{\rm RV} = \sqrt{2} \times 20$ m s^{-1} (including measurement errors and the intrinsic instability of stellar atmospheres). Hence, from equation (6), $\sigma_a = 8$ m s^{-1}. I focus on the case of $\chi^2_{\rm limit} = 4$ (their Fig. 6) corresponding to $a_{\rm thr} = \sqrt{8}\sigma_a$. They demand that the rms scatter of the astrometric measurements be less than 1 μas for a $T_m = 5$ yr mission, which corresponds to $\alpha_{\rm thr} = (720)^{1/2} \mu{\rm as}/T_m^2 = 1.07\ \mu{\rm as\ yr}^{-2}$, and place their grid stars at $D = 2$ kpc (S. Frink 2001, private communication). From equation (17), these values imply $K = 2.23$, and hence $G(K) = 0.67$. From equation (22), I find log $P_\dagger = 5.2$, which I then apply to their adopted binary distribution function $df_b/d \log P = 0.087 \exp[-(\log P - 4.8)^2/2 \times 2.3^2]$, to find $df_b/d \log P_\dagger = 0.086$. Equation (18) then predicts that 4.3% of their original sample should be found to be accelerators, or since 25% of that sample was rejected by their RV selection, 4.3%/0.75 = 5.8% of the grid stars. This compares with the value 5.5 ± 0.5% shown in their Figure 6, i.e., in excellent agreement.

7. SUMMARY OF FORMULAE

I find that the fraction of grid-star or reference-star candidates that are eliminated by RV surveillance is equal to the fraction with binary companions having $P < P_*$, where P_* is given by equations (10) and (11), in terms of mean mass of the companion distribution and the threshold of acceleration detection, $a_{\rm thr} = N\sigma_a$. Here σ_a is the acceleration measurement error, given by equation (6). (I used $N = 2$.) There is a small correction for eccentricity given explicitly by equation (13), and of course some candidates will be falsely rejected because of statistical fluctuations, depending on the choice of N.

The fraction of initial candidates that survive RV surveillance but nevertheless have detectable astrometric accelerations is given by equation (18), $f_{\rm acc} = (3/4)(df_b/d \log P_\dagger)G(K)$. The first term is simply the differential binary distribution evaluated at $P_\dagger$, which is given explicitly by equation (22). The second term is given by equation (21), $G(K) = \log[K + (K^2 + 1)^{1/2}]$, as a function of K, which

[3] In the case of ParaSIM, which measures relative offsets in only one direction, two reference stars must be found in each of two orientations. I ignore explicit consideration of ParaSIM here, but it is straightforward to extend the results presented for this case.

characterizes the ratio of RV to astrometric sensitivities and is given by equation (17). In this case the correction for eccentricity is negligible.

I find that a relatively modest RV survey can remove all but $\sim 5\%$ of grid-star candidates that will go on to show detectable accelerations, and all but $\sim 0.4\%$ of those that will show detectable jerks. I argue that it is only the latter very small fraction that must be eliminated from the grid.

For reasons summarized in § 5.2, reference-star selection is much more demanding than grid-star selection. I find that to have a 95% chance of ultimately locating two reference stars (the minimum required to be sensitive to distant planetary companions of the target star), eight candidates must be initially surveyed for each target star.

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